Simulation of Groundwater Mound Perching in Layered Media

Arland D. Schneider, James N. Luthin
MEMBER ASAE FELLOW ASAE

ABSTRACT

FORMATION of perched groundwater mounds was analyzed through numerical analysis of coupled saturated-unsaturated soil water flow. The unsteady soil water flow equation was solved by the Alternating Direction Implicit (ADI) numerical technique with iteration to remove the nonlinearity. In the numerical model, hydraulic conductivities, recharge rates, and flow domain geometery were varied to observe the important parameters affecting the formation of perched water table. The rate of formation of perched water tables depended primarily on the recharge rate and the saturated hydraulic conductivity of a semi-pervious, subsurface layer. If the ratio of the recharge rate to this hydraulic conductivity was less than 10, a perching condition did not exist. When the ratio ranged from 10 to 25, the perching condition was weak. When the ratio was greater than 25, the perching condition was strong.

INTRODUCTION

Semi-pervious soil strata are often a major constraint to artificially recharging groundwater formations from surface infiltration structures. Even in highly permeable media, one or more thin layers can seriously reduce percolation of water through the soil profile. Groundwater mounds that perch on semi-pervious layers can rise to the soil surface and limit the infiltration rate. Recharge rates are then determined primarily by the hydraulic conductivity of a subsurface layer rather than by the infiltration rate at the soil surface.

Theoretical analyses and field investigations of perched water tables beneath recharge structures are limited. Marmion (1962) initiated the theoretical study of perched mounds with a steady-state analysis that was verified with a viscous-flow model. Steady flow occurred when leakage through the semi-pervious stratum equaled the recharge rate. Brock (1976) developed a numerical, Dupuit-Forchheimer solution for unsteady perched mounds. Agreement between the numerical results and sand box model data was good, except when the mounds were high and narrow. For mounds perched on a thick, semi-pervious stratum, Khan et al. (1976) developed the potential and stream functions for two-dimensional and axisymmetric three-dimensional flow. The solution is more rigorous than Dupuit-Forchheimer theory, but the thick perching layer often does not represent field conditions. The analyses that are applicable to thin perching layers use the Dupuit-Forchheimer assumptions and do not consider unsaturated flow.

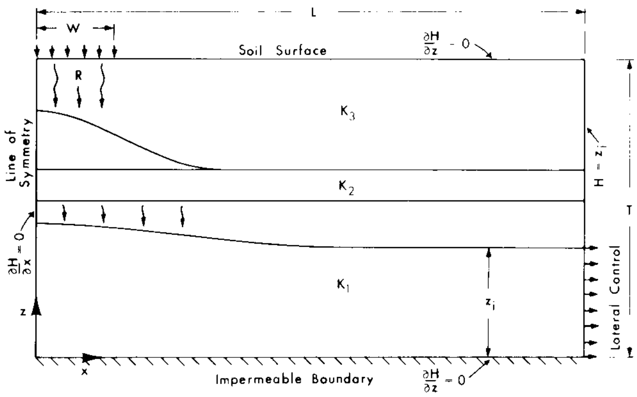

FIG. 1 Flow domain for a three-layered soil-aquifer system.

The objective of this research was to predict the formation of perched groundwater mounds through numerical analysis of coupled saturated-unsaturated soil water flow. The soil and underlying aquifer were modeled as a single flow domain with continuous flow from the unsaturated zone to the saturated zone.

MATHEMATICAL MODEL

The problem is to describe water flow through a three-layered soil-aquifer system, in which the hydraulic conductivity of the thin, center layer is low enough to form a perched water table (Fig. 1). Steady recharge at a rate, R, occurrs from a basin of width 2 W. Flow is symmetrical about the basin center so only half the flow domain must be considered. Since the basin length is much greater than the width, flow is two-dimensional. The percolating water perches above layer 2, leaks through the slowly permeable layer and forms a second groundwater mound at the original water table. The hydraulic gradient caused by the percolating water causes saturated and unsaturated flow to a vertical discharge surface referred to as the lateral control. The height of the perched mound decreases to zero above layer 2, and the height of the groundwater mound must be zero at x = L.

Boundary conditions for the flow problem are illustrated in Fig. 1. The line of symmetry and the impermeable boundary are streamlines so the gradient across these boundaries is zero. Along the soil surface the flux is R for $x \leqslant W$ and zero for $x > W$. The distant boundary has a constant hydraulic head, z_i, so flow may occur in both the saturated and unsaturated zones. The initial condition for the model was drainage to equilibrium,

Article was submitted for publication in June 1977; reviewed and approved for publication by the Soil and Water Division of ASAE in December 1977.

Contribution from the USDA-SEA, and the Water Science and Engineering Dept., University of California at Davis, in cooperation with The Texas Agricultural Experiment Station, Texas A&M University.

The authors are: ARLAND D. SCHNEIDER, Agricultural Engineer, USDA Southwestern Great Plains Research Center, Bushland, TX; JAMES N. LUTHIN, Professor, Water Science and Engineering Dept., University of California, Davis.

This article is reprinted from the TRANSACTIONS of the ASAE (Vol. 21, No. 5, pp. 920, 921, 922, 923, and 930, 1978)
Published by the American Society of Agricultural Engineers, St. Joseph, Michigan

when the unsaturated thickness was 100 cm or less or drainage to a minimum negative pressure head when the unsaturated thickness was greater than 100 cm.

Since the formation of groundwater mounds includes only soil wetting, hysteresis was not considered. The equations proposed by Gardner (1958) were used to uniquely relate the hydraulic conductiviey and soil water content to the negative pressure head of the soil:

$$K = \frac{K_o}{1 - A_k h^3} \quad \text{[1]}$$

$$\theta = \frac{\theta_o}{1 - A_w h^3} \quad \text{[2]}$$

where

K = hydraulic conductivity
K_o = saturated hydraulic conductivity
θ = soil water content
θ_o = saturated soil water content
h = negative soil water pressure head

and A_k and A_w are coefficients relating the hydraulic conductivity and soil water content to the negative soil water pressure head.

The partial differential equation describing soil water flow in two dimensions is:

$$\frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial H}{\partial z}\right) = \frac{\partial \theta}{\partial t} \quad \text{[3]}$$

where x and z are the spatial variables, t is the time variable, and $H = p/\gamma + z = h + z$.

Equation [3] is complicated by the presence of two dependent variables. The chain rule of partial derivatives can be used to transform the time derivative:

$$\frac{\partial \theta}{\partial t} = \frac{\partial \theta}{\partial h}\frac{\partial h}{\partial H}\frac{\partial H}{\partial t} \quad \text{[4]}$$

The term, $\partial\theta/\partial h$, is the specific water capacity, C, and $\partial h/\partial H = 1$.

Specific water capacity is obtained by differentiating equation [2]:

$$C = \frac{-3\, A_w \theta_o h^2}{(1 - A_w h^3)^2} \quad \text{[5]}$$

After transforming the time derivative and adding a source term, the soil water flow equation becomes:

$$\frac{\partial}{\partial x}\left(K\frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial z}\left(K\frac{\partial H}{\partial z}\right) = C\frac{\partial H}{\partial t} - Q \quad \text{[6]}$$

where Q is the source per unit area per unit time within the flow domain.

Equation [6] is a strongly-nonlinear, parabolic partial differential equation with no known analytic solution. The equation was solved by the Alternating Direction Implicit (ADI) technique with iteration to remove the nonlinearity.

The ADI method proposed by Peaceman and Rachford (1955) and by Douglas and Peaceman (1955) employs two finite difference equations over time steps of $\Delta t/2$. The first equation is implicit only in one direction, and the second equation is implicit only in the opposite direction. Each row or each column in the grid is swept to compute an intermediate solution at the $n + 1/2$ time step. The grid is then swept in the opposite direction to compute the true solution at the $n + 1$ time step. The unknowns in each difference equation are limited to three; thus, each system of equations can be efficiently solved by the recursive form of Gaussian elimination.

For iterative solution of nonlinear partial differential equations Rubin (1968) modified the ADI equations to advance the solution a full time step for sweeping in each direction. His first ADI equation uses known values to approximate the solution at the end of a full time step. The second ADI equation uses the approximated values to improve the solution at the end of the full time step. Taylor (1974) recommended the Rubin version of the ADI method because it generally requires fewer iterations in the solution of nonlinear equations. When Rubin's equation is applied to equation [6], the ADI equations for sweeping in the z and x directions, respectively, are:

$$\frac{\partial}{\partial x}\left(K^{n+1,2m}\frac{\partial H^{n+1,2m}}{\partial x}\right) + \frac{\partial}{\partial z}\left(K^{n+1,2m}\frac{\partial H^{n+1,2m+1}}{\partial z}\right) =$$

$$C_{i,j}^{n+1/2,2m}\frac{H_{i,j}^{n+1,2m+1} - H_{i,j}^{n}}{\Delta t} - Q_{i,j}^{n+1} \quad \text{[7]}$$

$$\frac{\partial}{\partial x}\left(K^{n+1,2m}\frac{\partial H^{n+1,2m+2}}{\partial x}\right) + \frac{\partial}{\partial z}\left(K^{n+1,2m}\frac{\partial H^{n+1,2m+1}}{\partial z}\right) =$$

$$C_{i,j}^{n+1/2,2m}\frac{H_{i,j}^{n+1,2m+2} - H_{i,j}^{n}}{\Delta t} - Q_{i,j}^{n+1} \quad \text{[8]}$$

where i, j, and n are the indices in the x, z, and t directions, respectively, and m is an iteration index.

An iteration parameter is required in equations [7] and [8] to insure convergence and reduce the number of iterations. The iteration parameter proposed by Douglas et al. (1959) and modified for the soil water flow equation by Rubin (1968) was used in this research. It is defined as $I_m = \overline{R}^p$ where I_m varies cyclically with the iteration cycles. The range of $\overline{R}$ is $0.20 \leqslant \overline{R} \leqslant 0.35$, and $p = 1, 2, \ldots, \overline{S}$ with $\overline{S}$ varying from 4 to 6. Values of $\overline{R}$ and $\overline{S}$ are selected by trial and error to achieve the fastest convergence. To use the iteration parameter, the terms

$$\overline{K}_n I_m \left(H_{i,j}^{n+1,2m+1} - H_{i,j}^{n+1,2m}\right)$$

$$\text{and } \overline{K}_n I_m \left(H_{i,j}^{n+1,2m+2} - H_{i,j}^{n+1,2m+1}\right)$$

are added to the right side of equations [7] and [8], respectively. The value of $\overline{K_n}$ is:

$$\overline{K}_n = K_{i-1/2,j}^{n} + K_{i+1/2,j}^{n} + K_{i,j-1/2}^{n} + K_{i,j+1/2}^{n}$$

Groundwater Mound Perching

(Continued from page 923)

pervious layer. The hydraulic conductivity of the capillary fringe was many times as large as the initial hydraulic conductivity of the drained soil. In some simulations, horizontal flow in the unsaturated zone accounted for 15 to 25 percent of infiltration from the recharge basin. For this reason, maximum heights of the perched mounds were not accurately predicted.

DISCUSSION

The R/K_2 ratios are guidelines for selecting surface water spreading sites for groundwater recharge. Since K_2 is a saturated hydraulic conductivity, it can be obtained through field measurements or by core measurements. The surface infiltration rate can be obtained with surface infiltration measurement techniques.

A large reduction in hydraulic conductivity is required to develop the $R/K_2 > 10$ condition for the formation of perched groundwater mounds. For a perching condition to develop, the surface soil will normally be 50 or more times as permeable as the semi-pervious layer. Such large variations in the hydraulic conductivity could be detected with geologic logging or permeability measurements of cores. Both techniques are commonly used to select groundwater recharge spreading areas.

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In the development of equations [7] and [8], the time derivative of equation [6] was replaced by a forward difference approximation. Finite differencing of the spatial derivatives gives the ADI finite difference equations for sweeping in the z and x directions, respectively:

$$\frac{K^{n+1,2m}_{i+1/2,j}\left(H^{n+1,2m}_{i+1,j} - H^{n+1,2m}_{i,j}\right)}{(1/2)(\Delta x_L + \Delta x_R)\Delta x_R} - \frac{K^{n+1,2m}_{i-1/2,j}\left(H^{n+1,2m}_{i,j} - H^{n+1,2m}_{i-1,j}\right)}{(1/2)(\Delta x_L + \Delta x_R)\Delta x_L} +$$

$$\frac{K^{n+1,2m}_{i,j+1/2}\left(H^{n+1,2m+1}_{i,j+1} - H^{n+1,2m+1}_{i,j}\right)}{(1/2)(\Delta z_B + \Delta z_T)\Delta z_T} - \frac{K^{n+1,2m}_{i,j-1/2}\left(H^{n+1,2m+1}_{i,j} - H^{n+1,2m+1}_{i,j-1}\right)}{(1/2)(\Delta z_B + \Delta z_T)\Delta z_B} =$$

$$\frac{C^{n+1/2,2m}_{i,j}\left(H^{n+1,2m+1}_{i,j} - H^{n}_{i,j}\right)}{\Delta t} + \bar{K}_n I_m \left(H^{n+1,2m+1}_{i,j} - H^{n+1,2m}_{i,j}\right) - Q^{n+1}_{i,j}$$

. . . . [9]

$$\frac{K^{n+1,2m}_{i+1/2,j}\left(H^{n+1,2m+2}_{i+1,j} - H^{n+1,2m+2}_{i,j}\right)}{(1/2)(\Delta x_L + \Delta x_R)\Delta x_R} - \frac{K^{n+1,2m}_{i-1/2,j}\left(H^{n+1,2m+2}_{i,j} - H^{n+1,2m+2}_{i-1,j}\right)}{(1/2)(\Delta x_L + \Delta x_R)\Delta x_L} +$$

$$\frac{K^{n+1,2m}_{i,j+1/2}\left(H^{n+1,2m+1}_{i,j+1} - H^{n+1,2m+1}_{i,j}\right)}{(1/2)(\Delta z_B + \Delta z_T)\Delta z_T} - \frac{K^{n+1,2m}_{i,j-1/2}\left(H^{n+1,2m+1}_{i,j} - H^{n+1,2m+1}_{i,j-1}\right)}{(1/2)(\Delta z_B + \Delta z_T)\Delta z_B} =$$

$$\frac{C^{n+1/2,2m}_{i,j}\left(H^{n+1,2m+2}_{i,j} - H^{n}_{i,j}\right)}{\Delta t} + \bar{K}_n I_m \left(H^{n+1,2m+2}_{i,j} - H^{n+1,2m+1}_{i,j}\right) - Q^{n+1}_{i,j}$$

. . . . [10]

The new symbols in equations [9] and [10] are:

$K_{i-1/2,j} = (1/2)(K_{i,j} + K_{i-1,j})$; $K_{i,j-1/2} = (1/2)(K_{i,j} + K_{i,j-1})$

$K_{i+1/2,j} = (1/2)(K_{i+1,j} + K_{i,j})$; $K_{i,j+1/2} = (1/2)(K_{i,j+1} + K_{i,j})$

$\Delta x_L = x_i - x_{i-1}$; $\Delta z_B = z_j - z_{j-1}$

$\Delta x_R = x_{i+1} - x_i$; $\Delta z_T = z_{j+1} - z_j$

Schneider (1976) presented a detailed derivation of the Rubin version of the ADI equations for solving equation [6].

The numerical model was verified with the sand box model data and the numerical Dupuit-Forchheimer solution of Brock (1976). Agreement between the numerical model and the Dupuit-Forchheimer solution was good (Schneider, p. 73, 1976). Brock considered his solution accurate, except for thick mounds with large vertical flow components. Both the numerical model and Brock's numerical solution consistently underestimated the sand box measurements. We believe the horizontal unsaturated flow in the numerical model was too large and this caused the model to underestimate the sand box measurements.

In the numerical model, the recharge rate and hydraulic conductivities were varied to provide a range of perching conditions. The selected hydraulic conductivities gave $K_1 \leq K_3$ and $K_3 >> K_2$. As a result a perched groundwater mound was caused by a single slowly permeable layer. The selected recharge rates gave $0.2 < R/K_3 < 0.4$ and $10 < R/K_2 < 40$. In the analyses to be reported, L = 10W, and, T, the flow domain thickness ranged from 140 to 800 cm.

RESULTS

Formation of groundwater mounds above the semi-pervious layer and at the water table is illustrated by the two groups of nested curves in Fig. 2. A perched groundwater mound will be refereed to as a "perched mound," and the groundwater mound at the water table will be referred to as the "groundwater mound." Initially, the perched mound did not exist, and the water table remained steady as water percolated through the unsaturated soil.

The perched mound formed after 0.43 days of recharge and rose rapidly as the semi-pervious layer limited vertical soil water movement. After 0.56 days of recharge, the perched mound was wide as the recharge basin, and after 1.42 days, saturated flow was nearly continuous between the recharge basin and the perched mound.

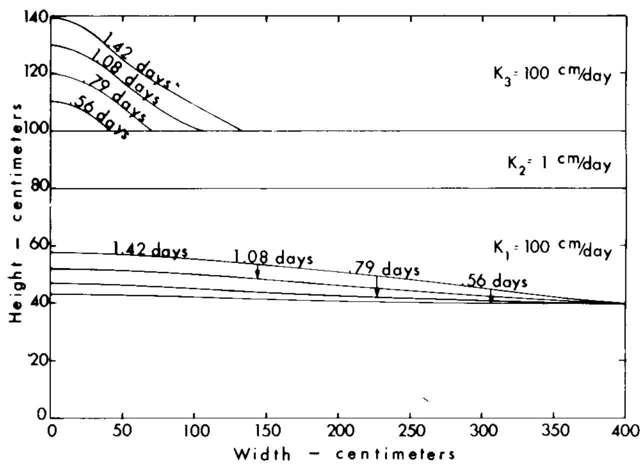

FIG. 2 Formation of groundwater mounds with W = 40 cm and R = 40 cm/day.

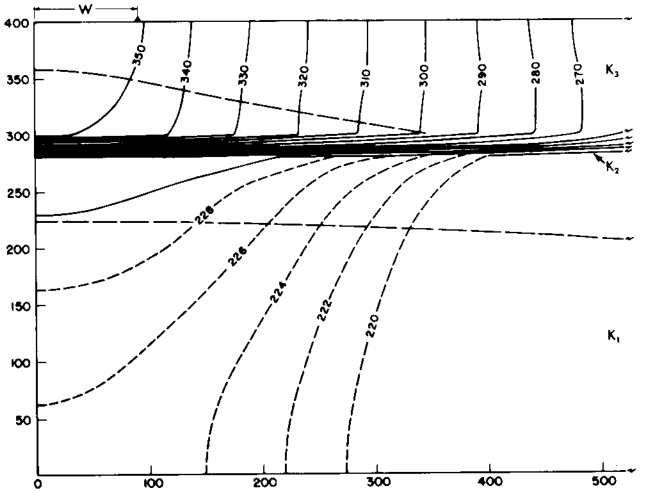

FIG. 3 Potential lines in a layered soil aquifer system as flow approached steady state, W = 90 cm and R = 100 cm/day. The potential lines are labeled as total head in centimeters.

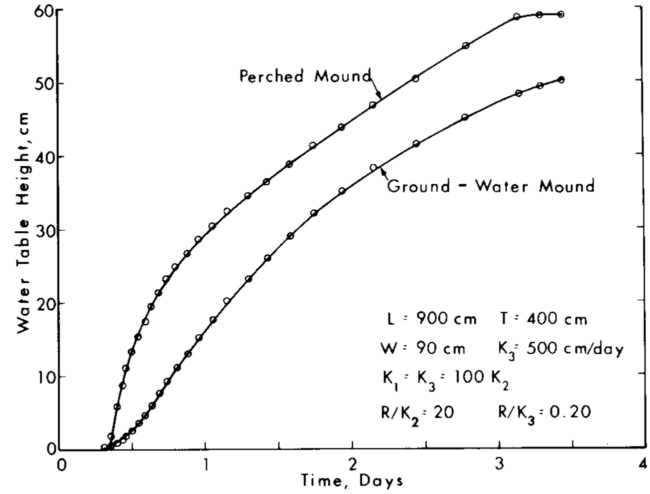

FIG. 4 The rate of formation of a perched mound above the semi-pervious layer and the underlying groundwater mound above the original water table.

When leakage through the semi-pervious layer approached the mass recharge rate from the basin, the perched mound slowly approached steady state.

The dominant influence of the semi-pervious layer on soil water flow is illustrated by the potential lines in Fig. 3. Flow is approaching steady state, and positions of the perched mound and the groundwater mound are sketched. For the perched mound, transition from vertical to essentially horizontal flow occurs within a distance 2W of the basin center. Soil water flow beneath the recharge basin is mainly vertical, but at the basin edge, the horizontal components is greater than the vertical component. Closely spaced potential lines within the semi-pervious layer show the large head loss across the perching layer. Since flow is approaching steady state, leakage through the semi-pervious layer is essentially equal to the mass recharge rate from the basin. A second transition from vertical to horizontal flow occurs beneath the semi-pervious layer. Dashed, supplementary potential lines beneath the water table show a low gradient zone typical of deep groundwater formations.

The rate of formation of a perched mound and a groundwater mound is illustrated in Fig. 4 for the soil-water flow parameters shown in the same figure. The perching condition was not strong, and both mounds grew at about the same rate. The perched mound reached quasi-steady state at 3.3 days, while the groundwater mound was still rising. The rate of formation of perched mounds depended primarily on the recharge rate and the hydraulic conductivity of the semi-pervious layer. The two parameters were combined by calculating R/K_2, the ratio of the recharge rate to the saturated hydraulic conductivity of the semi-pervious layer.

The time after recharge began at the soil surface for a perched mound to form is plotted in Fig. 5 as a function of R/K_2. Flow domain geometry was similar for all data points in the figure. Both the recharge rate and the hydraulic conductivity ratio, K_3/K_2, varied, while obtaining the range of R/K_2. The linear relationship between the two parameters shows the importance of R/K_2 in determining the conditions for perched mounds.

Three conditions for perched water tables were observed depending on the R/K_2 ratio. For smaller values of R/K_2, a perched water table did not form. Larger values of R/K_2 defined either weak or strong perching conditions.

A perching condition did not exist for $R/K_2 < 10$, and a second water table never formed. The pressure head at the top of the slowly-permeable layer increased to small negative values, but flow never became saturated.

When R/K_2 ranged from 10 to 25, a weak perching condition existed. The perched water table formed slowly after the wetting front passed the semi-pervious layer. When the groundwater mound began to form, the soil profile beneath the recharge basin had been wetted to small negative soil water pressures. If the unsaturated zone was thin, the groundwater mound began to form soon after the perched mound formed.

For $R/K_2 > 25$, a strong perching condition existed. A perched water table formed rapidly after the wetting front reached the semi-pervious layer. Perched mounds also formed before appreciable flow occurred through the semi-pervious layer. Thus, most flow through the semi-pervious layer was in the saturated phase. Even with thin flow domains, the perched mound formed before there was an appreciable rise of the groundwater table.

The model flow domains were small in comparison to prototype groundwater recharge systems. As a result, unsaturated flow was more important than in field size flow systems. When a perched mound formed, a second zone of nearly saturated soil occurred above the semi-

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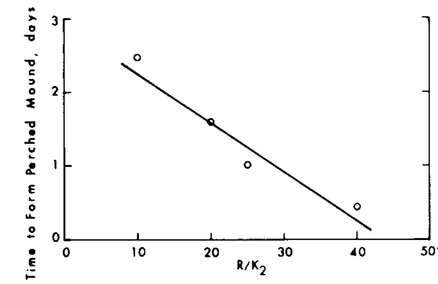

FIG. 5 The effect of R/K_2 on the rate of formation of a perched mound, L = 400 cm and T = 140 cm.